

Energy Movements in the Medium separating Electrified or Gravitating Particles

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DISCUSSION.

A letter from Dr. E. H. BARTON was read emphasizing the necessity of taking into account the damping in the oscillator-train, and at the same time pointing out why, in his opinion, the corrections applied by Mr. Yule were inadequate and failed to yield intelligible results.

Prof. RÜCKER congratulated Mr. Yule on his work, and on the importance of the results he had obtained.

XXXIV. *Energy Movements in the Medium separating Electrified or Gravitating Particles.* By H. N. ALLEN, *University of Nebraska, Lincoln, Neb.**

1. FARADAY and Maxwell have shown that it is possible to look on the potential energy of electric separation as residing in the surrounding dielectric, and that each of the cells, bounded by the walls of a tube of force and two neighbouring equipotential surfaces, can be looked upon as containing a certain definite amount of energy.

This energy-distribution is not in general permanent, and can only be regarded as a step towards some simpler arrangement. Thus a positive and a negative electrified body suspended in space, and acted on only by the electrical forces between them, are never in equilibrium until they are in actual contact. The energy-distribution in the dielectric changes constantly as they approach. Poynting has shown † how energy is transferred from one point to another in an electromagnetic field; and we are quite accustomed to think of energy as flowing from dynamo to motor through the æther, or from primary to secondary in an alternating-current transformer.

In the following pages an attempt is made to deduce a few of the consequences of Maxwell's suggestion with regard to

* Read March 8, 1895.

† Phil. Trans. 1884, Pt. II. p. 343.

energy-distribution. The changes needed in the theory in order to apply it to gravitation are also indicated.

2. If two equal charges of positive and negative electricity $+M$ and $-M$ are separated by a distance l , the equation to the curve in which the equipotential surface V intersects the plane of the paper is

$$\frac{M}{\sqrt{y^2 + (\frac{1}{2}l - x)^2}} - \frac{M}{\sqrt{y^2 + (\frac{1}{2}l + x)^2}} = V, \quad (1)$$

the origin being the point midway between the particles, and the X-axis being the line joining this point to the positive particle.

3. The number of tubes of force proceeding from a charge M has sometimes been taken as M but more generally as $4\pi M$. For graphical methods the former plan seems most convenient, and in order to avoid confusion it is proposed that the resulting tubes should be called "tubes of polarization," while the smaller tubes retain the name "tubes of induction." The polarization, as defined by J. J. Thomson in his 'Recent Researches in Electricity and Magnetism,' is measured by the number of these polarization tubes which pass through a square centimetre perpendicular to their direction.

Using Maxwell's method of drawing the boundary lines between the tubes of polarization, the equation to the n^{th} line in the case mentioned above will be

$$\frac{\frac{1}{2}l - x}{\sqrt{y^2 + (\frac{1}{2}l - x)^2}} + \frac{\frac{1}{2}l + x}{\sqrt{y^2 + (\frac{1}{2}l + x)^2}} = 2 - \frac{2n}{M}, \quad (2)$$

where the straight line drawn from the positive to the negative particle is called zero, and that drawn in the opposite direction M .

4. The point of intersection of the equipotential surface V , the line of force n , and the plane of the paper lies on the curves (1) and (2). If, then, l in these equations be regarded as a variable parameter, they will together represent the curve along which the corner of an energy-cell moves, when the two particles come together. To plot this curve, the equipotential surfaces and lines of force might be drawn for a number of different distances between the particles, and corresponding points of intersection joined.

The path of the energy-cell can be obtained with less labour from measures made on a single diagram, drawn to represent the lines of force and equipotential surfaces about the two particles, when these are separated by a given distance. This is done by taking advantage of the following properties of these lines :—

1. When the distance between the particles is changed, corresponding tubes of polarization in the two diagrams are similar to one another.

2. The equipotential surfaces in the two diagrams are also similar to one another. If, however, the distance between the particles changes from l to al , the equipotential surface V changes to a similar surface, on which the potential is $\frac{V}{a}$.

If, then, the n^{th} line of force cuts the equipotential surface V at the point x, y in the first diagram, the n^{th} line will cut the equipotential surface $\frac{V}{a}$ at the point ax, ay in the second diagram.

Thus, in the case where $M=12$ and $l=10$ the coordinates of the following intersections, among others, were found by measurement on a carefully prepared diagram.

Line of force.	Equipotential surface.	x .	y .
7	1	5.15	5.95
7	4	5.35	2.28

If the diagram were enlarged in the ratio of 4 to 1, the coordinates of the point corresponding to the intersection of 7 and 4 would be $4x$ and $4y$ or 21.4 and 9.12.

The potential at this point would be $V' = \frac{V}{a} = 1$.

Thus we have found the coordinates of the point of intersection of the line of force 7 and the equipotential surface 1, when the distance between the particles is 40 centim.

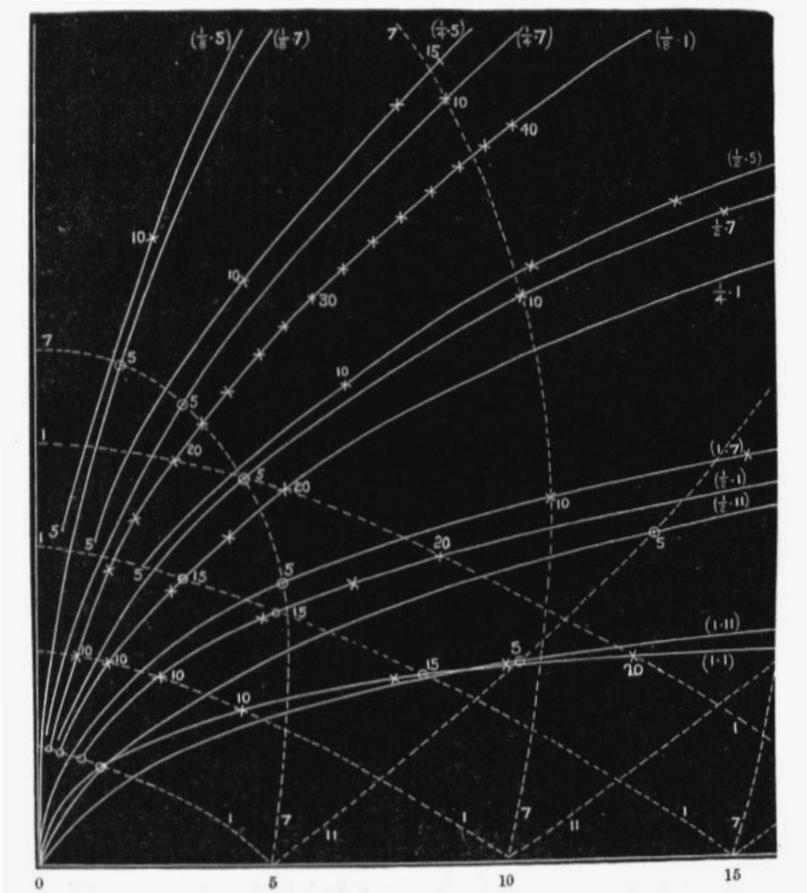
In the same way, and by means of the same original diagram, the point of intersection can be found for a number of other distances, and the path which it follows, when the two particles approach one another, can be plotted as in fig. 1.

In this figure the first of the numbers in brackets attached to each curve gives the number of the equipotential surface,

and the second that of the intersecting line of force. The numbers along the curves show the distance of the particles from the centre of gravity, when the energy-cell is at the curve.

It will be seen that the energy-cells move in more or less

Fig. 1.

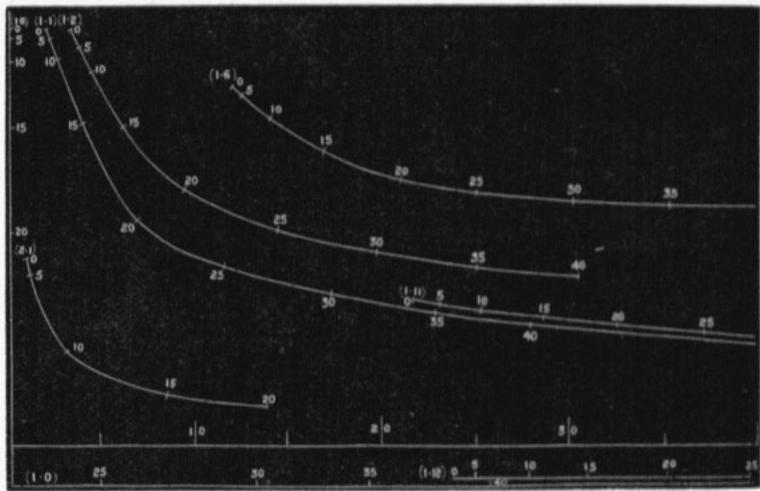


parabolic curves towards the centre of gravity of the two particles, and that during this process they are constantly approaching the two particles ; so that, if these are not infinitely small, energy must be constantly passing into them from the æther.

Maxwell has shown that in all cases the number of energy-cells in the æther is twice the potential energy of the system. So that, if we suppose each cell to contain half a unit of energy, we may regard them as preserving their identity when the bodies approach, and as being absorbed by the moving bodies, so that their potential energy is converted into kinetic. A conductor moves in such a way that the absorbed energy is a maximum.

Fig. 1 also illustrates the rotation of the energy-cell as the particles approach. If on any one of the energy-paths, as

Fig. 2.



for instance ($\frac{1}{2}$. 7), the direction of the line of force (indicated by a dotted line) cutting it at different points be noticed, it will be seen that this direction constantly changes as the particles approach, so that the tangent to the line of force rotates about the point of intersection. The velocity of the different energy-cells is indicated by crosses along the curves, at such points that, when the two particles come together from an infinite distance apart, the cells move from cross to cross in equal intervals of time. The same time is taken in moving from the cross marked 10 to the origin.

5. In fig. 2 the paths of the energy-cells are shown when two equal like particles repel one another. It will be noticed

that, as the particles move apart, the energy in each half of the field follows the motion of the particle in that half, and at the same time moves nearer to the central line.

It is clear that the result will be that energy will flow from the dielectric into the particle as it moves away, and being lost as potential energy will reappear as energy of motion. Even when the particles are an infinite distance apart there will still be a number of energy-cells in the æther, that is, the particles will still possess potential energy.

This is explained by supposing that every polarization tube must begin and end somewhere. In this case we may suppose that though A and B are an infinite distance apart, the negative particle C is again an infinite distance beyond A, and the negative particle D an infinite distance beyond B, so that the tubes from A end in C, and those from B in D. Then, after B and A have been repelled to a great distance from one another and all the potential energy due to their original nearness has been exhausted, there will still remain the energy due to the separation of D from B and of A from C.

6. If H_e is the intensity of the electrostatic field, or the force acting upon a particle with unit charge ;

B_e the number of tubes of induction per sq. centim., or the induction in the direction of the line of force ;

D_e the number of polarization tubes per sq. centim., or the polarization or displacement in the same direction ;

K the dielectric constant of the medium ;

we have $H_e = \frac{B_e}{K}$, $D_e = \frac{KH_e}{4\pi} = \frac{B_e}{4\pi}$.

The length of an energy-cell is $\frac{1}{H_e}$ or $\frac{K}{4\pi D_e}$.

The area of section of the cell is $\frac{1}{D_e}$.

Thus the volume of the cell is $\frac{K}{4\pi D_e^2}$.

The energy-density is $\frac{2\pi D_e^2}{K} = \frac{B_e^2}{8\pi K} = \frac{KH_e^2}{8\pi}$.

The volume of an energy-cell is proportional to the square of its length measured along a line of force. Figs. 1 and 2 show how in general the portion of the line of force intercepted between two equipotential surfaces diminishes as the

Fig. 3.

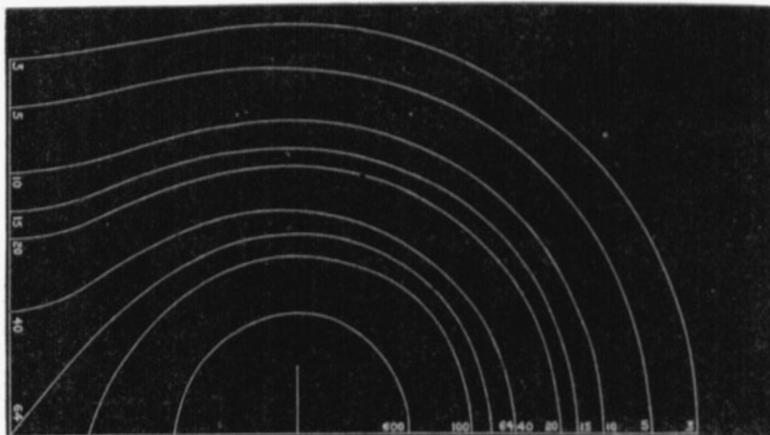
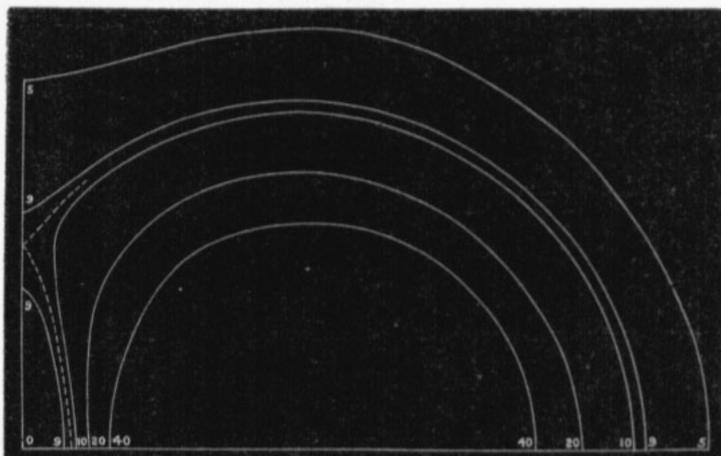


Fig. 4.



movement proceeds. The volume of the energy-cell constantly tends to diminish, though in some cases, as in fig. 2, the volume of some of the cells has to increase for a while to

allow the rest to contract. During this contraction the length of the cell bears to its area the constant ratio $\frac{K}{4\pi}$.

It has seemed worth while to trace roughly the curves of equal energy-density in the æther about equal unlike and equal like particles. These are shown in figs 3 and 4. The numbers apply to masses of 100 with a distance 10 between them.

7. Maxwell has discussed (Scientific Papers, vol. i. pp. 570-571) the modifications which must be made in the theory to make it fit in with the observed facts of gravitation.

The energy diagram for two gravitational particles is the same as that for two like electrified particles. The polarization tubes go off to infinity, and do not, as far as we know, end on negative gravitational matter.

The properties of the cells between the polarization tubes and the equipotential surfaces are exactly opposite in the two cases of electricity and gravitation.

The electrical energy-cells are constantly tending to contract both in length and section, but so that the area of the section always bears a constant ratio to the length of the cell. The gravitational energy-cells, on the other hand, constantly tend to expand, the same ratio holding between area and length as in the former case. While each electrical cell contains according to Maxwell's theory half a unit of energy, the gravitational cells must be supposed to contain half a unit less of energy than the same volume of undisturbed æther. The gravitational cells are negative energy-cells. This expansion of the negative energy-cells implies a kind of contraction of the energy of the gravitational æther. In the case of electricity the final state is reached when the energy-density is everywhere zero, all the energy having passed into the particles; in the case of gravitation when the density is uniform over a series of concentric shells, being a maximum at an infinite distance from ponderable matter.

8. Fig. 2 shows the path of the negative energy-cells, when two equal gravitational particles come together from an infinite distance apart. It can be seen that the cells retreat from the particles as these approach one another. There will be a greater number of negative cells in the æther when the par-

ticles are in contact than when they are at a distance. This is merely another way of saying that (positive) energy has passed from the æther into the particles.

The instantaneous position of the lines of energy-flow, for a given position of the moving gravitational particles, is something like fig. 2, the motion being away from the median plane and towards the line joining the two particles. The flow of energy is on the whole in the direction opposite to the motion of the particle. The potential energy absorbed by two gravitational particles, when they fall together, comes from the space between them.

The increase in the number of negative energy-cells, when two gravitating masses approach, is equal to twice the gain in kinetic, or loss in potential energy. The proof is the same as in the case of static electricity, due regard being paid to the signs of the quantities involved.

Thus the theory may be advanced that each of these negative energy-cells contains half a unit less of energy than the same volume of æther in which no such cells exist. An increase in number of these cells corresponds to a flow of energy from the gravitational æther into matter, where it is converted into kinetic energy.

Some difficulty may arise in special cases on account of the fact that in the case of gravitation the attracting mass has a volume distribution, instead of being confined to a surface like static electricity on a conductor. Each unit mass sends out a tube of polarization, whether it is on the surface or not, and a considerable proportion of the energy-cells exist inside the surface.

In the case of a single homogeneous sphere, at a distance from all other bodies, one sixth of the negative energy-cells are within the surface.

It is clear, then, that in general the energy is not removed from the æther at the instant the negative cell passes out through the surface, but perhaps rather at the instant that it comes out from the molecule. In other words, energy may be regarded as flowing in through the surface, but is only absorbed and converted into kinetic energy when it passes into a molecule.

9. Maxwell has calculated the enormous pressure along the

lines of force and the equal tension at right angles to them required, according to his theory, to account for the attraction of the earth by the sun. A minimum value for the amount of energy in one cubic centimetre of gravitational æther can be calculated by finding the negative energy-density on the surface of the sun considered as a body of uniform density.

The volume of the energy-cells at any point is $\frac{4\pi}{H_e^2}$.

The intensity of the gravitational field on the surface of the sun is $H_e = \frac{M}{r^2}$, where M is the mass and r the radius of this body. If ρ is the density of the sun the volume of an energy-cell is

$$\frac{4\pi r^4}{M^2} = \frac{4\pi r^4}{\left(\frac{4}{3}\pi\rho r^3\right)^2} = \frac{9}{4\pi\rho^2 r^2}.$$

The amount of energy lacking in each of these cells is half a unit. Hence one cubic centimetre of æther lacks $\frac{2}{3}\pi\rho^2 r^2$ units. These are static units of energy. One dyne equals 1.544×10^7 static units of force, therefore one erg equals 1.544×10^7 static units of energy. Take r as $6370000 \times 100 \times 110$ centim. and ρ as 1.39 . Then the density of negative energy at the surface of the sun will be

$$\begin{aligned} \frac{2}{9} \frac{\pi \times 1.39^2 \times 637^2 \times 110^2 \times 10^{12}}{1.544 \times 10^7} & \text{ ergs per c.c.} \\ & = 4.289 \times 10^{14} \text{ ergs per c.c.} \\ & = 42.89 \times 10^6 \text{ joules per c.c.} \\ & = 16 \text{ horse-power hours per c.c.} \end{aligned}$$

This would seem to mean that, at a distance from all gravitating matter, a cubic centimetre of æther contains at least this amount of energy.

10. That the theory may require an even greater energy-density than this is seen as follows. Suppose the energy of the gravitational æther to be due to vortex filaments or tubes of directed energy interlacing in every direction. Take at any point three axes at right angles to one another. The irregular distribution can be replaced in imagination by six equal sets of filaments parallel to the positive and negative directions of these three axes.

In space at a great distance from gravitational matter each of these sets of filaments will contain the same amount of energy. Let $6a$ be the amount of energy in one cubic centimetre of free æther. Let the positive direction of the X-axis at a point correspond with the positive direction of the gravitational line of force at that point. Let b be the volume of the gravitational energy-cell. Then five sets of filaments in the cell may contain $5ab$ units of energy, and the sixth, which has the direction of the X-axis, $ab-1$ units. Altogether the cell will contain $6ab-1$ units, while the same volume at a distance from gravitational matter will contain $6ab$ units of energy.

This would require the existence of 96 or say 100 horsepower hours in every cubic centimetre.

11. It is an interesting question whether this theory of energy-distribution in gravitational æther is or is not the simplest and most probable. That other distributions are conceivable appears evident. For example, the æther along the straight line joining the centres of two bodies might be regarded as a stretched elastic cord, the laws of contraction being of course quite different from those which hold for ordinary elastic bodies.

Another explanation which seems possible in the case of two gravitational particles is as follows. In the energy diagram for two equal electrified particles with opposite sign, suppose each infinitesimal tube of polarization to be divided into two equal tubes with opposite directions. If they are regarded as vortex filaments, each vortex will work in with its neighbours rotating the other way, like one friction-pulley on another.

Then it is clear that there is no way in which we can regard the two particles as opposed in sign, and yet if each of the two sets of vortex filaments acts as electrostatic tubes always act, the particles will be attracted together.

The energy-flow in this case will be as indicated in fig. 1, and the distribution of energy-density as in fig. 3. It has not been found possible to map the energy field in this manner for three or more particles. Indeed, the difficulties are such that it seems improbable that this method can be applied.

12. In conclusion it may be well to notice again the

assumption on which this paper is based : that the energy-cells preserve their identity, and carry the same energy with them throughout their path. It is not clear that this is necessarily true. It may be that energy passes from the potential into the kinetic form in the æther itself, and not merely on the surface of the molecules. Kinetic energy may consist of the motion of the whole system of energy-cells. This would lead us very near the theory which regards the molecule as being nothing but the mathematical centre from which forces proceed, or perhaps, from another point of view, as having infinite extension.

XXXV. *On a Simple Form of Harmonic Analyser.* By
GEORGE UDNY YULE, *Demonstrator in Applied Mathematics,*
University College, London *.

“The subject of the decomposition of an arbitrary function into the sum of functions of special types has many fascinations. No student of mathematical physics, if he possess any soul at all, can fail to recognize the poetry that pervades this branch of mathematics.”—OLIVER HEAVISIDE.

§ 1. ABOUT a year ago several instruments for determining the coefficients of a Fourier Series expressing the equation to a given curve were described before this Society by Professor Henrici †. One of them, Professor Henrici's shifting-table analyser, used a planimeter as the integrator; an arrangement that seemed to me very noteworthy from the point of view of simplicity and cheapness. The analyser I am going to describe also uses a planimeter: consequently it can also only give the value of one coefficient at a time.

§ 2. Let PQR be the curve to be analysed. Let the base PR range from $x = -l$ to $x = +l$, and the equation to the

* Read March 8, 1895.

† *Ante*, p. 77; also Catalogue of the Mathematical Exhibition at Munich (1892-93).